Oberlin College
CSCI 383
Fall 2026
Lecture 25. Beyond Diagonalization
Kozen - Sections 32 and 33

Last Time:

- Membership problem is not recursive.
- Halting problem is not recursive.

Def: A property P is *decidable* if $\{x \mid P(x)\}$ is recursive (and undecidable if not).

Def: A property P is *semi-decidable* if $\{x \mid P(x)\}$ is recursively enumerable.

Non-RE Languages

What's the difficulty with MP?

- Easy to determine if a TM accepts.
- Easy to determine if a TM halts and rejects.
- Problem: Determining if a TM loops.

Define $\overline{\text{MP}} = \{M\#x \mid M \text{ rejects or loops on } x\}$

Assume $\overline{\text{MP}}$ was RE.

Give a total TM for MP.

- Run TMs for MP, $\overline{\text{MP}}$ in parallel.
- Accept if first TM accepts, reject if second TM accepts.

Conclusion: $\overline{\text{MP}}$ is not RE.

(Neither is $\overline{\text{HP}}$, by a similar argument.)

MP via Other Problems

Language: $L_\epsilon = \{M \mid M \text{ encodes a TM that accepts } \epsilon\}$.

Why isn't this trivial? TM might bounce around writing a bunch of stuff without accepting.

Is this r.e.? (Yes.)

Which is an easier problem? MP or L_ϵ?

If you had a total TM for MP, could use it as a total TM for L_ϵ.

We'll prove other direction: Given total TM for L_ϵ, could make total TM for MP.

Assume you had a total TM M for L_ϵ.

Goal: Produce a total TM M^* that takes $N\#x$ and outputs whether N accepts x.

Two subroutines:

- M
- "reprogramming"

Reprogramming subroutine R:

- Input: $N\#x$
- Output: "reprogrammed" encoding of a TM N' which does the following on input y
  - Simulates N on x.
  - If N accepts x, accept y.
  - If N rejects x, reject y.

Equivalent in Python: look for the return statement in N, add an if statement.

Questions:

- If N accepts x, what is $L(N')$?
- If N rejects x, what is $L(N')$?
- If N loops on x, what is $L(N')$?

To solve MP on $N\#x$

- Feed $N\#x$ into R to get N'.
- Input N' to M.
- Return result.

Note: We never run N or N'. All we're doing is diddling the code.

- If N accepts x, what does M output?
- If N rejects x, what does M output?
- If N loops on x, what does M output?