Oberlin College
CSCI 383
Fall 2026
Lecture 24. Analyzing TMs
Kozen - Section 32

Last time: Universal TMs.

Can encode TMs as strings.

MP = $\{N\#x \mid N$ encodes a TM that accepts $x\}$ is recursively enumerable

How:

- Checked to make sure N was properly encoded.
- Simulated N on x.

Today: Other problems on encoded TMs.

(We'll take proper encoding for granted: can always check at beginning of your algorithm.)

Round 1

$L_1 = \{N \mid N$ has 100 states$\}$

Recursive! Can read off of N's description.

$L_2 = \{N \mid N$ accepts $\epsilon\}$

Recursively enumerable: Just simulate N on ϵ using universal TM.

Is there a clear place to stop?

$L_3 = \{N \mid N$ halts on ϵ in the first 100 steps$\}$

Recursive: Just simulate N on ϵ using a universal TM. After 100 steps, see if we've halted.

Round 2

$L_4 = \{N\#q \mid N$ enters state q on input $\epsilon\}$

Recursively enumerable: Just simulate N on ϵ using a universal TM, and accept if we ever enter q.

$L_5 = \{N \mid N$ accepts some string$\}$
,
Recursively enumerable: Just simulate N on every string using timesharing.

$L_6 = \{N \mid N$ halts on every string in ≤ 2 steps$\}$

Recursive: N halts on all strings in 2 steps if and only if it halts on all zero- or one-character string in two steps.

So just simulate N for ≤ 2 steps on all zero- or one-character strings.

Round 3

$L_7 = \{N \mid N \text{ revisits the same state multiple times before halting}\}$

Recursive: After $|Q| + 1$ steps, must either revisit or halt.

$L_8 = \{N \mid N \text{ halts on some string in 100 steps}\}$

Recursive: N halts on some string in ≤ 100 steps if and only if it halts on some string of length ≤ 99 in 100 steps.

$L_9 = \{N \mid N \text{ moves its read head 100 steps to the right on } \epsilon\}$
Configurations: What information summarizes state of machine?

- Contents of tape. ($|\Gamma|^100$ options)
- Location of read/write head. (100 options)
- State. ($|Q|$ options)

Observations:

- If N re-visits a configuration, it loops.
- Let n be the number of configurations where the TM hasn't gone past first 100 characters.

  If we run for $n + 1$ steps without going 100 to the right, we are guaranteed to re-visit a configuration.

$n = 100|Q|(|\Gamma|)^{100}$.

Round 4

$L_{10} = \{N_1 \# N_2 \mid L(N_1) \cap L(N_2) \neq \emptyset\}$

$L_{11} = \{N \mid L(N) = \Sigma^*\}$

$L_{12} = \{N \# x \mid x \notin L(N)\}$