Oberlin College
CSCI 383
Fall 2025
Lecture 2. Deterministic Finite Automata
Kozen - Sections 3 and 4

Last Time:

- Strings (Standard input for computational problems)
- Languages (Yes instances)

Today:

- Language operations
- DFAs informally
- Formal definitions

Kleene Star Again

Recall definition(s) from last time.

Let $A = \{\texttt{ab}, \texttt{ac}\}$ and $B = \{\texttt{b}\}$ What are

- the seven shortest strings in A^* (ε, `ab`, `ac`, `abab`, `abac`, `acab`, `acac`)
- the seven shortest strings in $(AB)^*$ (ε, `abb`, `acb`, `abbabb`, `abbacb`, `acbacb`, `acbabb`)
- A^*B^* the eight shortest strings in (ε, `b`, `ab`, `ac`, `bb`, `abb`, `acb`, `bbb`)

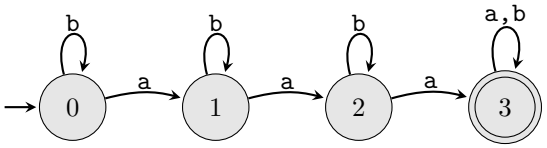
Deterministic Finite Automata

Idea: Computation via flowcharts.

Alphabet: $\Sigma = \{\texttt{a}, \texttt{b}\}$.

Computer will read in string $x \in \Sigma^*$, output "Accept" or "Reject".

Draw DFA



Place pebble at start state.

Read one character, transition.

Etc.

Accept if and only if I end in accept state.

Example: `aabaabba`.

What're some example strings that will be accepted?

How would you describe the set of strings that will be accepted?

Varying the graph structure will yield different sets.

Formal Definition: A *deterministic finite automaton* (DFA) is a quintouple $M = (Q, \Sigma, \delta, s, F)$:

- Q is the *state space* (finite set)
- Σ is the input alphabet (finite set)
- s is the *start state*
- $F \subseteq Q$ is the set of *accept states*.
- $\delta : Q \times \Sigma \to Q$ is the *transition function*.

(Draw analogies to the picture.)

Note: The picture is often called the *transition diagram*.

Informal definition of acceptance: Follow the transition function from s. If you end up in F, accept. Else, reject.

Def: The *language* $L(M)$ of a DFA M is the set of strings accepted by M.

A language A is *regular* if there exists some DFA M for which $L(M) = A$.

Exercise: Given the following formal definition, build the transition diagram.

- $Q = \{0, 1, 2\}$
- $\Sigma = \{\mathtt{a}, \mathtt{b}\}$
- $\delta(0, \mathtt{a}) = 1$, $\delta(0, \mathtt{b}) = 2$, $\delta(1, \mathtt{a}) = 0$, $\delta(1, \mathtt{b}) = 2$, $\delta(2, \mathtt{a}) = 2$, $\delta(2, \mathtt{b}) = 2$
- $s = 0$
- $F = \{1\}$

  What's the language of this DFA? ($\{\mathtt{a}^k \mid k \text{ odd}\}$)