CS 301: Languages and Automata **Spring 2018**

Homework 6

Due: Sunday, May 06, 2018

Instructions

This assignment is due Sunday, May 06, 2018 at 11:59PM (Central Time). Solutions must be submitted on Gradescope. Your solutions must be typeset. Handwritten solutions will not be graded and will receive a 0.

Late submissions will be accepted within 24 hours after the deadline with a penalty of 25% of the assignment grade. No late submissions will be accepted more than 24 hours after the deadline.

Problem 1 [10 points] Prove that A is decidable if and only if $A \leqslant_m 0^*1^*$. [*Hint: Show that if A is decidable, then $A \leqslant_m 0^*1^*$ and if $A \leqslant_m 0^*1^*$, then A is decidable.*]

Problem 2 [15 points] Let $\varepsilon_{TM} = \{\langle M \rangle \mid M \text{ is a TM and } M \text{ accepts } \varepsilon\}$.

Prove that $A_{TM} \leqslant_m \varepsilon_{TM}$ by constructing a TM that takes $\langle M, w \rangle$ as input and outputs $\langle M' \rangle$ such that M accepts w if and only if M' accepts ε.

Problem 3 Prove that class P is closed under:

- **a.** [**5 points**] **Union**. [*Hint: Let M_1 and M_2 be deterministic TMs that decide languages A and B in polynomial time. Construct a new deterministic TM M that decides $A \cup B$ in polynomial time.*]
- **b.** [**5 points**] **Complement**. [*Hint: Let M be a deterministic TM that decides language A in polynomial time. Construct a new deterministic TM M' that decides $\bar{A}$ in polynomial time.*]

Problem 4 [10 points] Prove that class NP is closed under **union**. [*Hint: Let M_1 and M_2 be nondeterministic TMs that decide languages A and B in polynomial time. Construct a new nondeterministic TM M that decides $A \cup B$ in polynomial time. Alternatively, let M_1 and M_2 be deterministic TMs that verify languages A and B in polynomial time. Construct a new deterministic TM M that verifies $A \cup B$ in polynomial time.*]

Problem 5 [10 points] A **cycle** in a directed graph is a path that starts and ends at the same vertex.

Let $CYCLE = \{\langle G, v \rangle \mid G \text{ is a directed graph that has a cycle starting at vertex } v\}$.

Prove that $CYCLE \in P$. [*Hint: Construct a deterministic TM that decides $CYCLE$ in polyonimial time and show that your construction is correct.*]

Problem 6 [15 points] A **triangle** in an undirected graph is a 3-clique.

Let $TRIANGLE = \{\langle G \rangle \mid G$ is an undirected graph that has a triangle $\}$.

Prove that $TRIANGLE \in P$. [*Hint: Construct a deterministic TM that decides $TRIANGLE$ in polyonimial time and show that your construction is correct.*]

Problem 7 [15 points] A **dominating set** S is a subset of vertices of a graph G such that every vertex not in S is adjacent to at least one vertex in S.

Let $DOMINATING\text{-}SET = \{\langle G, k \rangle \mid G$ is a graph that has a dominating set with k vertices$\}$.

Prove that $DOMINATING\text{-}SET \in NP$. [*Hint: Construct a nondeterministic TM that decides $DOMINATING\text{-}SET$ in polyonimial time and show that your construction is correct. Alternatively, construct a deterministic TM that verifies $DOMINATING\text{-}SET$ in polyonimial time.*]

Problem 8 [15 points] Two graphs G and H are said to be **isomorphic** if the vertices of G may be reordered so that G is identical to H.

Let $ISO = \{\langle G, H \rangle \mid G$ and H are isomorphic graphs$\}$.

Prove that $ISO \in NP$. [*Hint: Construct a nondeterministic TM that decides ISO in polyonimial time and show that your construction is correct. Alternatively, construct a deterministic TM that verifies ISO in polyonimial time.*]