CS 301: Languages and Automata — Spring 2018

Homework 5

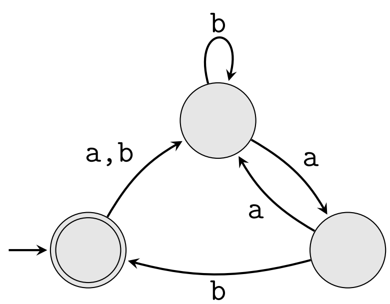
Due: Sunday, April 22, 2018

Instructions

This assignment is due Sunday, April 22, 2018 at 11:59PM (Central Time). Solutions must be submitted on Gradescope. Your solutions must be typeset. Handwritten solutions will not be graded and will receive a 0.

Late submissions will be accepted within 24 hours after the deadline with a penalty of 25% of the assignment grade. No late submissions will be accepted more than 24 hours after the deadline.

Problem 1 Answer the following questions about the following DFA M and give reasons for your answers.



a. [**2 points**] Is $\langle M, \texttt{abaab}\rangle \in A_{\mathrm{DFA}}$?

b. [**2 points**] Is $\langle M, \texttt{bab}\rangle \in A_{\mathrm{DFA}}$?

c. [**2 points**] Is $\langle M\rangle \in A_{\mathrm{DFA}}$?

d. [**2 points**] Is $\langle M\rangle \in E_{\mathrm{DFA}}$?

e. [**2 points**] Is $\langle M, M\rangle \in EQ_{\mathrm{DFA}}$?

Problem 2 Closure properties of decidable languages.

a. [**5 points**] Prove that the class of decidable languages is closed under concatenation. [*Hint: Let M_1 and M_2 be TMs that decide languages A and B. Construct a new TM M to decide $A \circ B$. TM M will take as input some string $w = w_1 w_2 \cdots w_n$ where each $w_i \in \Sigma$ and will have to divide w into two pieces $x = w_1 w_2 \cdots w_k$ and $y = w_{k+1} w_{k+2} \cdots w_n$ for some $0 \leqslant k \leqslant n$ and check that $x \in A$ and $y \in B$. Make sure that M tries all $n+1$ possible x and y.*]

b. [10 points] Prove that the class of decidable languages is closed under Kleene star. [*Hint: Let M_1 decide language A. Construct a new TM M to decide A^*. Recall that for string w to be in A^*, there must be a division of w into k pieces $w = w_1 w_2 \cdots w_k$ for some $k \geqslant 0$ such that each $w_i \in A$. TM M will have to try all possible divisions for all values of k up to some number. If $|w| = n$, think about which values of k the TM M needs to consider. This problem is tricky!*]

Problem 3 Closure properties of Turing-recognizable languages.

a. [10 points] Prove that the class of Turing-recognizable languages is closed under concatenation. [*Hint: This is similar to the previous problem but now you have the issue that the TMs M_1 or M_2 may not halt on some division of w into x and y, but* will *halt and accept on some other division. Have M first write down all of the possible splits and then simulate M_1 and M_2 on each of the possible x and y in "parallel" by performing one step of the simulation of each TM at a time.*]

b. [15 points] Prove that the class of Turing-recognizable languages is closed under Kleene star. [*Hint: Use the hints for Problems 2b and 3a.*]

Problem 4 [10 points] Let $A = \{\langle R, S\rangle \mid R$ and S are regular expressions such that $L(R) \subseteq L(S)\}$. Prove that A is decidable by giving a TM that decides it. [*Hint: Your TM should construct one or more DFAs and use a decider for a language we've already shown to be decidable as a subroutine.*]

Problem 5 [10 points] Let $B = \{\langle R\rangle \mid R$ is a regular expression describing a language containing at least one string w that has `111` as a substring (i.e., $w = x\texttt{111}y$ for some x and y)$\}$. Prove that B is decidable by giving a decider for it. [*Hint: Your TM should construct one or more DFAs and use a decider for a language we've already shown to be decidable as a subroutine.*]

Problem 6 [10 points] Prove that EQ_{CFG} is undecidable. [*Hint: Give a reduction from ALL_{CFG}.*]

Problem 7 [10 points] Let $C = \{\langle M\rangle \mid M$ is a TM that accepts $w^{\mathcal{R}}$ iff it accepts $w\}$. Prove that C is undecidable. [*Hint: Give a reduction from A_{TM}.*]

Problem 8 [10 points] Consider the problem of determining whether a Turing machine M on an input w ever attempts to move its head left when its head is on the left-most tape cell. Formulate this problem as a language and show that it is undecidable.