CS 301: Languages and Automata **Spring 2018**

Homework 4

Due: Sunday, April 01, 2018

Instructions

This assignment is due Sunday, April 01, 2018 at 11:59PM (Central Time). Solutions for Part I must be submitted on Blackboard and solutions for Part II must be submitted on Gradescope.

Late submissions will be accepted within 24 hours after the deadline with a penalty of 25% of the assignment grade. No late submissions will be accepted more than 24 hours after the deadline.

Part I: JFLAP Constructions

Problem 1 [**50 points**] For each of the following languages, construct a Turing machine in JFLAP, version 7, that decides the language. To receive full credit for each language, you must submit three files: (1) the JFLAP file (e.g., `1a.jff`); (2) a text file (e.g., `1a-accept.txt`) with five strings that are in the language, one per line; and (3) a text file (e.g., `1a-reject.txt`) with five strings that are *not* in the language, one per line. In total, you should have 9 files.

Note that there is no explicit reject state in JFLAP. We assume that there is a transition to the reject state whenever a state lacks an outgoing transition for a particular symbol.

a. $A = \{w \mid w \in \{\texttt{0}, \texttt{1}\}^*$ contains twice as many 0s as 1s$\}$

b. $B = \{w \mid w \in \{\texttt{0}, \texttt{1}\}^*$ does not contain an equal number of 0s and 1s$\}$

c. $C = \{w\#w\#w \mid w \in \{\texttt{0}, \texttt{1}\}^*\}$

Part II: Proofs

Remember, your solutions to Part II must be typeset. Handwritten solutions will not be graded and will receive a 0.

Problem 1 Let A, B, and C be languages such that A is context free, B is regular, and C is context free.

a. [**5 points**] Prove that the difference between A and B, $A \setminus B$, is context free.

b. [**5 points**] Is the difference between A and C, $A \setminus C$, also context free? Justify your answer.

Problem 2 [**15 points**] Let $D = \{\mathtt{a}^n\mathtt{b}^m\mathtt{c}^k \mid n, m, k \geqslant 0 \text{ and } m \leqslant \mathtt{min}(n, k)\}$. Prove that D is not context free.

Problem 3 [**10 points**] Give an implementation-level description of a Turing machine that decides the language $E = \{x\#y\#z \mid x, y, z \in \{\mathtt{0}, \mathtt{1}\}^* \text{ are binary numbers and } x + y = z\}$.

For example, $\mathtt{101}\#\mathtt{11}\#\mathtt{1000}$ is in E because $\mathtt{101} + \mathtt{11} = \mathtt{1000}$ (in base 10, $5 + 3 = 8$), while $\mathtt{101}\#\mathtt{11}\#\mathtt{111}$ is *not* in E because $\mathtt{101} + \mathtt{11} \neq \mathtt{111}$ (in base 10, $5 + 3 \neq 7$).

Problem 4 [**15 points**] A *Turing machine with doubly infinite tape* is similar to an ordinary Turing machine, but its tape is infinite to the left as well as to the right. The tape is initially filled with blanks except for the portion that contains the input. Computation is defined as usual except that the head never encounters an end to the tape as it moves leftward. Show that this type of Turing machine recognizes the class of Turing-recognizable languages.