CS 301: Languages and Automata **Spring 2018**

Homework 1

Due: Sunday, February 04, 2018

Instructions

This assignment is due Sunday, February 04, 2018 at 11:59PM (Central Time). Solutions for Part I must be submitted on Blackboard and solutions for Part II must be submitted on Gradescope.

Late submissions will be accepted within 24 hours after the deadline with a penalty of 25% of the assignment grade. No late submissions will be accepted more than 24 hours after the deadline.

Part I: DFAs and NFAs

Construct a DFA/NFA in JFLAP that recognizes each of the following languages. To receive full credit for each language, you must submit a text file with a set of 5 strings that are in the language, a text file with a set of 5 strings that are not in the language, and a JFLAP file with the DFA/NFA.

Problem 1 Construct a DFA that recognizes each of the following languages:

a. **[10 points]** $A = \{w \in \{\mathtt{a}, \mathtt{b}\}^* \mid w$ starts with `a` and has odd length or starts with `b` and has even length$\}$.

b. **[10 points]** $B = \{w \in \{\mathtt{a}, \mathtt{b}\}^* \mid w$ does not contain the substring `ab` and does not contain the substring `ba`$\}$.

c. **[10 points]** $C = \{w \in \{\mathtt{0}, \mathtt{1}\}^* \mid w$ is a binary number divisible by 3$\}$. [*Hint: for example,* `1001` *(9 in base 10) is divisible by 3 so it would be accepted, while* `1010` *(10 in base 10) is not divisible by 3 so it would be rejected. Leading* `0`*'s (e.g.,* `001010`*) are permitted and the empty string is taken as a representation for* `0`.]

Problem 2 Construct an NFA that recognizes each of the following languages:

a. **[10 points]** $D = \{w \in \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}^* \mid |w| \geqslant 2$ and w starts and ends with the same symbol$\}$.

b. **[10 points]** $E = \{xy \mid x \in \{\mathtt{0}, \mathtt{1}\}^*$ and $y \in \{\mathtt{0}, \mathtt{1}\}^*$ and the length of x is at most 5 and every odd position of y is a `1`$\}$.

Part II: Closure Properties of Regular Languages

Solve each of the following problems. To receive full credit for each problem, you must justify why your construction is correct.

Problem 1 In class, we showed that if M is a DFA that recognizes language A, swapping the accepting and nonaccepting states yields a new DFA recognizing the complement of A. We then concluded that the class of regular languages is closed under *complement*.

a. **[10 points]** Show, by giving an example, that if N is an NFA that recognizes language B, swapping the accepting and nonaccepting states in N does not necessarily yield a new NFA that recognizes the complement of B.

b. **[5 points]** Is the class of languages recognized by an NFA closed under *complement*? Explain your answer.

Problem 2 **[15 points]** Prove that the class of regular languages is closed under *intersection*. That is, show that if A and B are regular languages, then $A \cap B = \{w \in A \text{ and } w \in B\}$ is also regular.

Problem 3 **[20 points]** Prove that the class of regular languages is closed under *reverse*. That is, show that if A is a regular language, then $A^{\mathcal{R}} = \{w^{\mathcal{R}} \mid w \in A\}$ is also regular. [*Hint: given a DFA* $M = (Q, \Sigma, \delta, q_0, F)$ *that recognizes* A*, construct a new NFA* $N = (Q', \Sigma, \delta', q_0', F')$ *that recognizes* $A^{\mathcal{R}}$*.*]