CS 301: Languages and Automata **Fall 2015**

Problem Set #3

Due: Tuesday, November 17, 2015

Problem 1 Show that any TM can be converted to one in which the head never attempts to move left on the left-most cell of the tape. [*Hint: Use a new tape symbol.*]

Problem 2 Prove that $L = \{\langle M \rangle \mid M \text{ is a DFA that accepts } w^{\mathcal{R}} \text{ whenever it accepts } w\}$ is decidable. [*Hint: Your decider should takes $\langle M \rangle$ as input and construct a new DFA M'. Then, it should use a decider for EQ_{DFA}.*]

Problem 3 Prove that a language L is decidable if and only if $L^{\mathcal{C}}$ is decidable.

Problem 4 Consider the problem of determining whether a computer program written in Python ever prints out "`Hello world!`" when run on some input w. Prove that this problem is undecidable. Formally, consider the language

$$HW = \{\langle P, w \rangle \mid P \text{ is a Python program that, on input } w, \text{ prints } \texttt{Hello world!}\}$$

and show that it is undecidable. [*Hint: Prove this by contradiction. Assume that R is a decider for HW. Build a new TM D that decides A_{TM} using R as a subroutine. Conclude that since A_{TM} is undecidable, this is a contradiction so HW must be undecidable.*]

Problem 5 Consider the problem of determining whether a TM M on input w ever attempts to move its head left when its head is on the left-most tape cell. Formulate this problem as a language and prove that it is undecidable. [*Hint: Use the result in problem 1 to build a new TM whose head only attempts to move left on the left-most cell of the tape when you want it to. Proceed similarly to problem 4.*]

Problem 6 Show that the class of Turing-recognizable languages is not closed under complement.

Problem 7 Consider the language

$$L = \{\langle M, w, q \rangle \mid M \text{ is a TM that when run on input } w \text{ never enters state } q\}.$$

If L is decidable, describe a TM that decides it. If L is not decidable, prove it.