CSCI 275: Programming Abstractions

Lecture 10: The world of folds

Fall 2024

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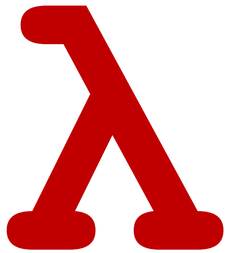
Slides gratefully borrowed from Molly Q Feldman



Questions for the good of the group?

α and β are types. And let's say `proc` takes elements of type α and produces elements of type β (i.e. the type of `proc` is $\alpha \rightarrow \beta$).

`When` calling `(map proc lst)`, what is the type of `lst`? What is the type of `map`'s return?

A. List of β, List of β
B. List of α, List of α
C. List of α, list of β
D. List of β, List of α
E. Something else

Review: map

Applies a procedure to each element of a list

α and β are types

`(map proc lst)`
`proc :` $\alpha \rightarrow \beta$
`lst : list of` α
`map` returns `list of` β

E.g.,

α `= number,` β `= integer`

```
(map floor '(1.3 2.8 -8.5))
```

Review: apply

Applies a procedure the arguments in a list

```
(apply proc lst)
```

proc : $\alpha_1 \times \alpha_2 \times \cdots \times \alpha_n \rightarrow \beta$
lst : (α_1 α_2 … α_n)
apply returns β

E.g.,
α_1 = number, α_2 = boolean, β = number

```
(apply (lambda (n b) (if b (- n) n))
       '(5 #t))
```

```
(define (fun lst)
  (cond [(empty? lst) base-case]
        [else (let ([head (first lst)]
                    [result (fun (rest lst))])
                (combine head result))]))
```

`lst`: list of α

`base-case`: β

What kind of function is `combine`?
(input type to output type)

A. `combine`: $\alpha \times \beta \to \alpha$

B. `combine`: $\alpha \times \beta \to \beta$

C. `combine`: $\beta \times \alpha \to \alpha$

D. `combine`: $\beta \times \alpha \to \beta$

Where we left off…..

Basic structure is the same!

```
(define (fun … lst)
  (cond [(empty? lst) base-case]
        [else
         (let ([head (first lst)]
               [result (fun … (rest lst))])
           (combine head result))]))
```

Function	base-case	(combine head result)
sum	0	(+ head result)
length	0	(+ 1 result)
map	empty	(cons (proc head) result)
remove*	empty	(if (equal? x head) result (cons head result))

Abstraction: fold right

`(foldr combine base-case lst)`

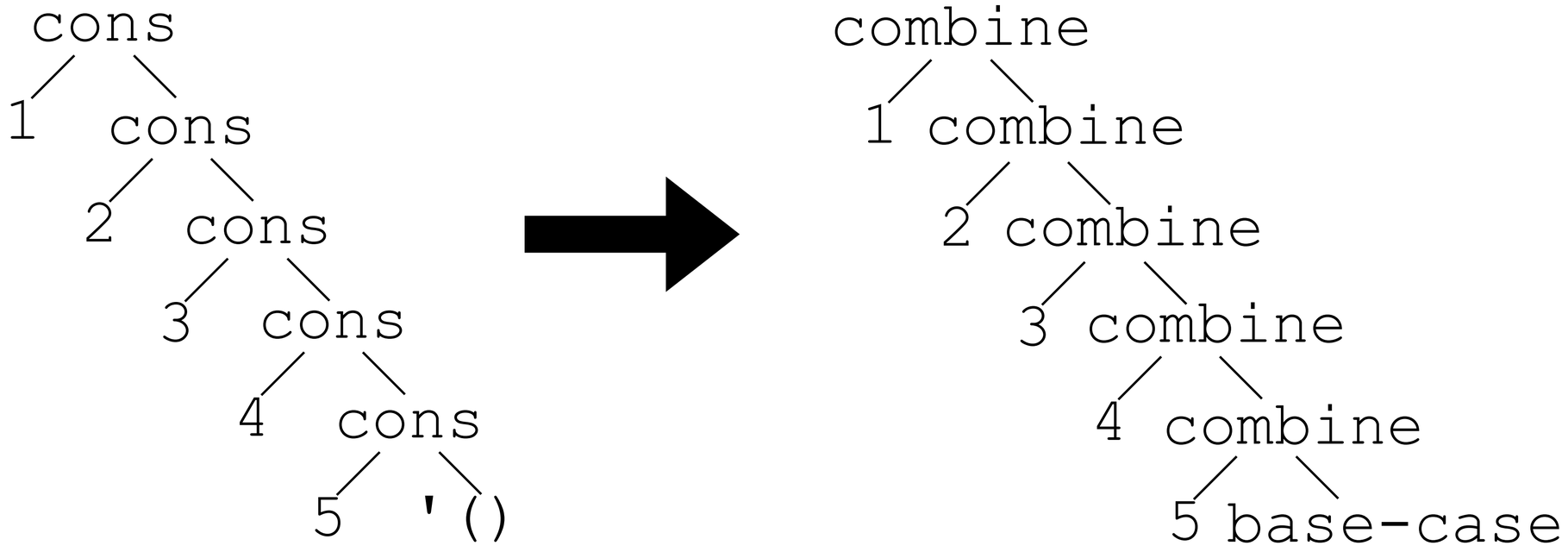
`combine`: $\alpha \times \beta \rightarrow \beta$
`base-case`: β
`lst`: list of α
`foldr`: $(\alpha \times \beta \rightarrow \beta) \times \beta \times (\text{list of } \alpha) \rightarrow \beta$

Elements of `lst` = $(x_1\ x_2\ \ldots\ x_n)$ and `base-case` are combined by computing

z_n = (combine x_n base-case)
z_{n-1} = (combine x_{n-1} z_n)
z_{n-2} = (combine x_{n-2} z_{n-1})
⋮
z_1 = (combine x_1 z_2)

Abstraction: fold right

`(foldr combine base-case lst)`



Possible input `lst`

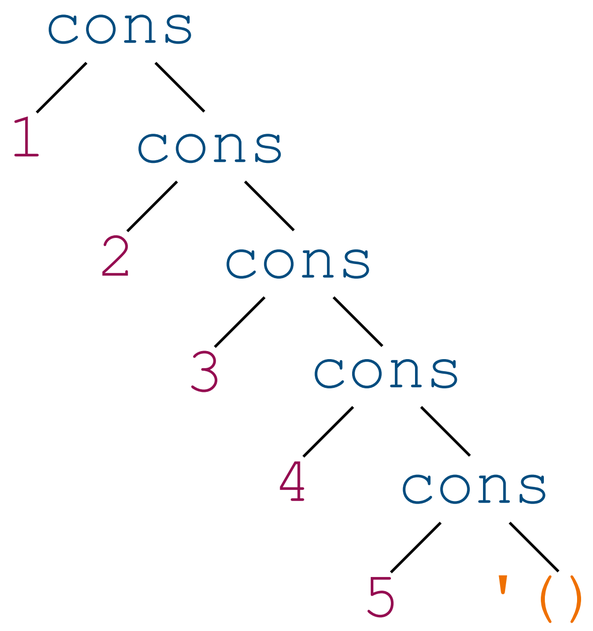
Executing `foldr`

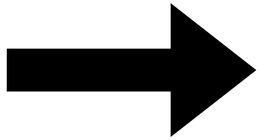
sum as a fold right

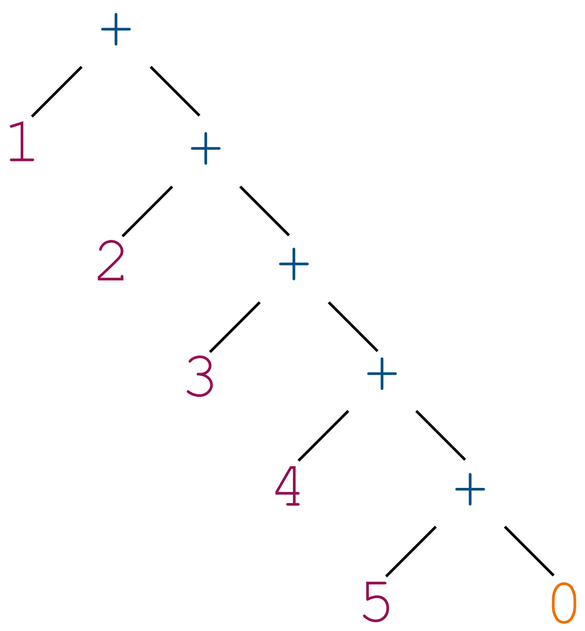
(foldr combine base-case lst)

```
(define sum
  (lambda (lst)
   (foldr + 0 lst)))
```

combine: number × number → number
base-case: number
lst: list of number



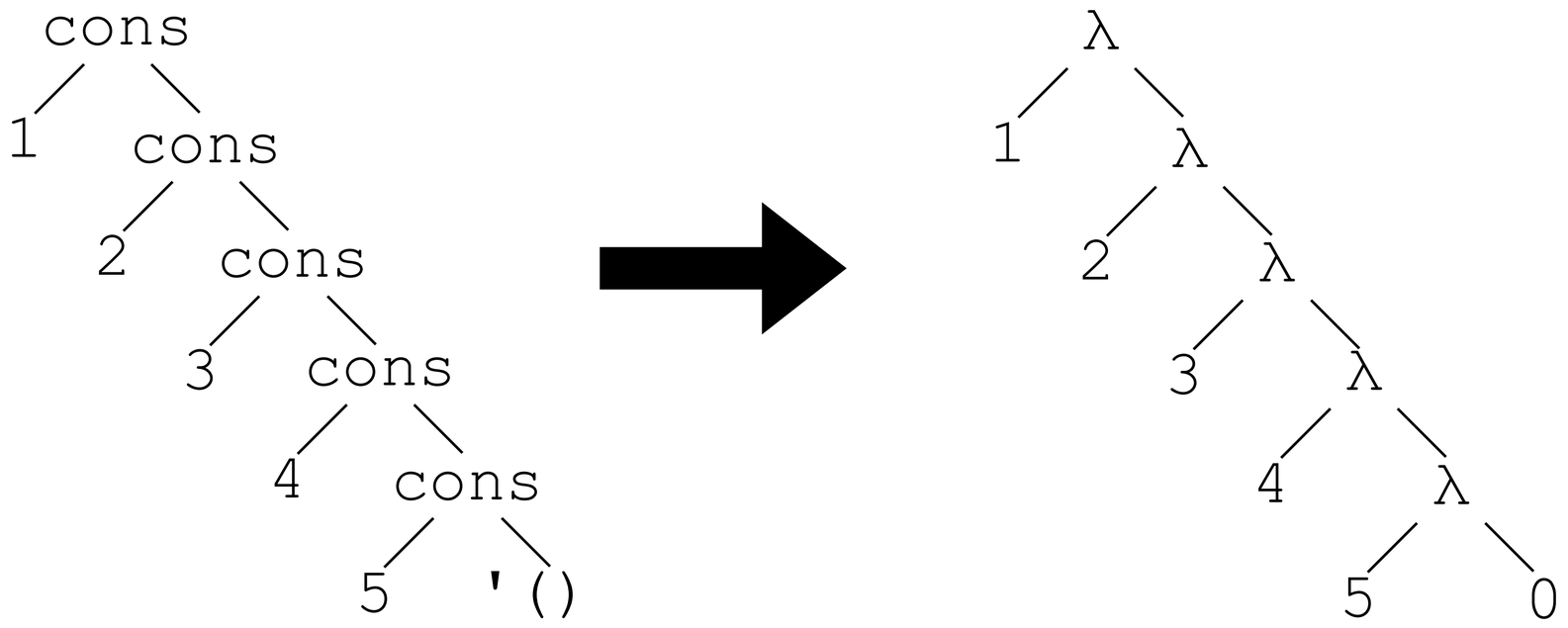




length as a fold right

(foldr combine base-case lst)

```
(define length
   (lambda (lst)
  (foldr (lambda (head result) (+ 1 result)) 0 lst)))
```



`map` as fold right

`(foldr combine base-case lst)`

```
(define (map proc lst)
  (foldr (lambda (head result)
           (cons (proc head) result))
         empty
         lst))
```

proc: $\alpha \rightarrow \beta$

combine: $\alpha \times$ (list of β) $\rightarrow$ list of β

base-case: list of β

lst: list of α

map: ($\alpha \rightarrow \beta$) $\times$ (list of α) $\rightarrow$ list of β

`remove*` as fold right

`(foldr combine base-case lst)`

```
(define (remove* x lst)
  (foldr (lambda (head result)
           (if (equal? x head)
               result
               (cons head result)))
         empty
         lst))
```

x: α

combine: $\alpha \times$ (list of α) $\rightarrow$ list of α

base-case: list of α

lst: list of α

remove*: $\alpha \times$ (list of α) $\rightarrow$ list of α

map: ($\alpha \rightarrow \beta$) $\times$ (list of α) $\rightarrow$ list of β

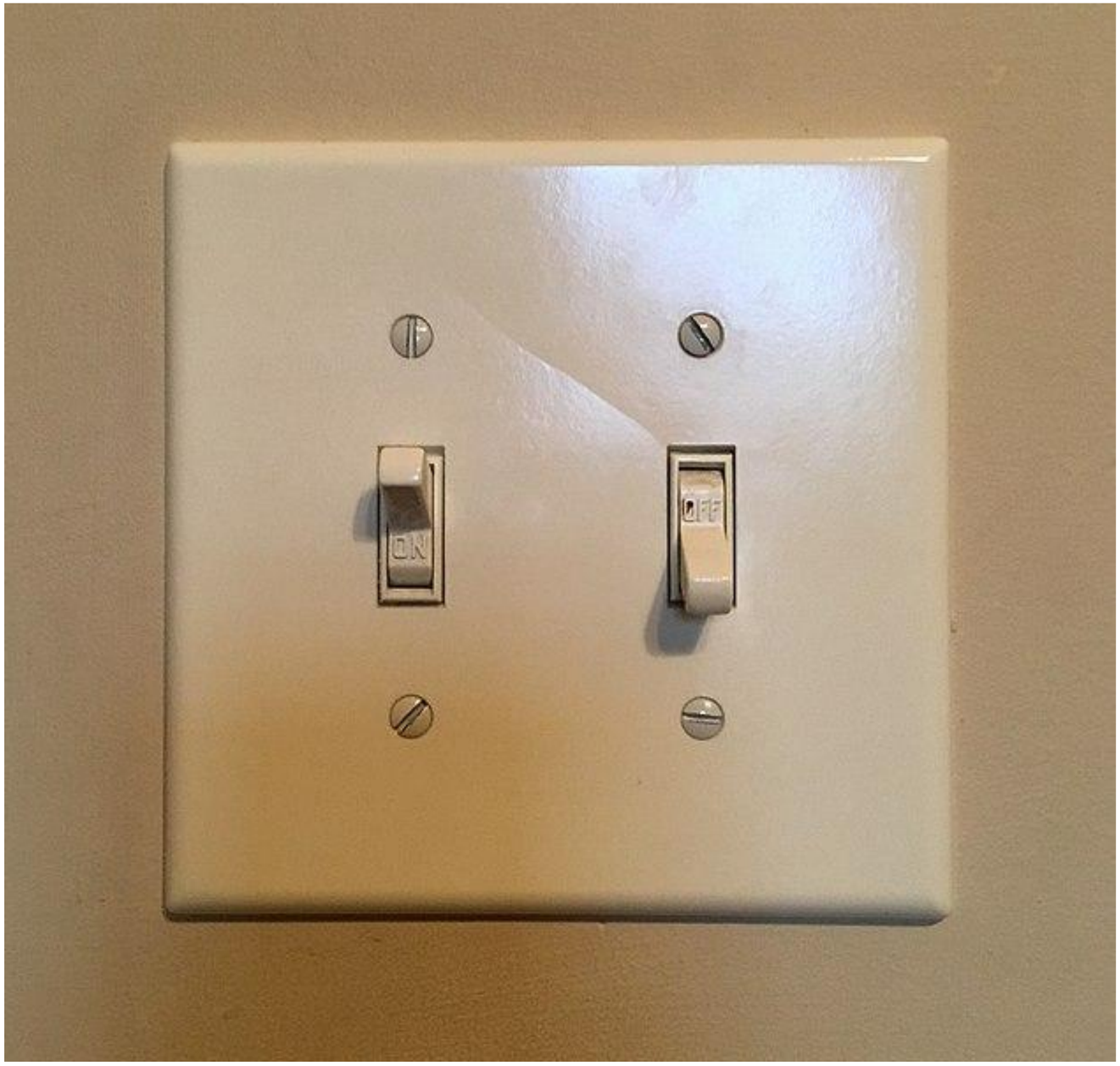
Consider the procedure

```
(foldr (lambda (str num)
         (+ num (string-length str)))
       0
       '("red" "green" "blue"))
```

What does this do?

A. Multiplies all the string lengths
B. Counts number of elements in the list
C. Sums all the string lengths
D. Error

Example: a light switch "state machine"



Example: a light switch "state machine"

Consider a light switch connected to a light

The light is in one of two states: on and off

- Represent this with symbols `'on` and `'off`

There are three actions we can take

- `'up`: move the switch to the up position; turns the light on
- `'down`: move the switch to the down position; turns the light off
- `'flip`: flip the position of the switch; changes the state of the light

If the light is initially `'off`, then after the sequence of actions `'(up up down flip flip flip)`, the light will be `'on`

Implement the state machine

Possible actions: `'up`, `'down`, `'flip`

Possible states: `'on`, `'off`

Write a `(next-state action state)` function that returns the next state of the light after the action is performed in the given state (no higher order needed!)

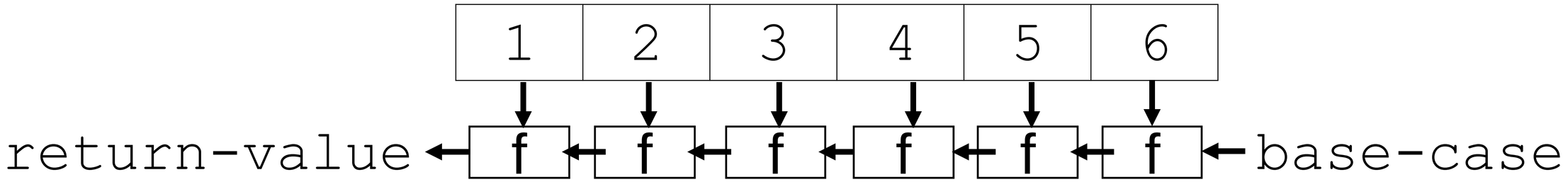
Write a `(state-after actions)` that returns the state of the light assuming it's initially `'off` and the actions in the list `actions` are performed in order

- Use `foldr!`
- Be careful about the order:

```
(state-after '(up flip)) => 'off
```

Takeaway from state machine example

`foldr` really is fold *right*



Next Up

Readings do continue!

Homework 2 is live, due Friday at 11:59pm via GitHub

- Feel free to use whatever structures you'd like to solve it (higher order not required, HW3/4 they will be!)

Weekly Reflection due Today

Summary Problems later today!