CS 241: Systems Programming
Lecture 13. Bits and Bytes 2

Spring 2020
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Internal data representation

Data are stored in binary

32 bit unsigned integer values are:

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```
00000000 00000000 00000000 00000000 = 0
```

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```
00000000 00000000 00000000 00000000 = 0
00000000 00000000 00000000 00000001 = 1
```

Internal data representation

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32 bit unsigned integer values are:

```
00000000 00000000 00000000 00000000 = 0
00000000 00000000 00000000 00000001 = 1
00000000 00000000 00000000 00000010 = 2
```

Internal data representation

Data are stored in binary

32 bit unsigned integer values are:

```
00000000 00000000 00000000 00000000 = 0
00000000 00000000 00000000 00000001 = 1
00000000 00000000 00000000 00000010 = 2
00000000 00000000 00000000 00000011 = 3
```

Internal data representation

Data are stored in binary

32 bit unsigned integer values are:

```
00000000 00000000 00000000 00000000 = 0
00000000 00000000 00000000 00000001 = 1
00000000 00000000 00000000 00000010 = 2
00000000 00000000 00000000 00000011 = 3
…
11111111 11111111 11111111 11111111 = 2³²-1
```

11111111 11111111 11111111 11111111 = $2^{32}-1$

Bitwise operators

Binary operators apply the operation to the corresponding bits of the operands
- ‣ `x & y` — bitwise AND
- ‣ `x | y` — bitwise OR
- ‣ `x ^ y` — bitwise XOR

Unary operator applies the operation to each bit
- ‣ `~x` — one's complement (flip each bit)

Boolean logic

A	B	~A	A&B	A|B	A^B
0	0	1	0	0	0
0	1	1	0	1	1
1	0	0	0	1	1
1	1	0	1	1	0

What is the value of `0x4E & 0x1F`?

A. 0xE

B. 0x51

C. 0x5F

D. 0xB1

E. 0xE0

Hex	Binary	Hex	Binary
0	0000	8	1000
1	0001	9	1001
2	0010	A	1010
3	0011	B	1011
4	0100	C	1100
5	0101	D	1101
6	0110	E	1110
7	0111	F	1111

Bit shifting

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Manipulates the position of bits

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- ‣ Left shift fills with 0 bits

Bit shifting

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‣ Left shift fills with 0 bits

‣ Right shift of unsigned variable fills with 0 bits

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- ‣ Right shift of signed variable fills with sign bit (Actually implementation defined if negative!)

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```
x << 2;  //  shifts bits of x two positions left
```

Bit shifting

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- ‣ Same as multiplying by 4

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x << 2;  //  shifts bits of x two positions left
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- ‣ Same as multiplying by 4

```
x >> 3;  //  shifts bits of x three positions right
```

Bit shifting

Manipulates the position of bits
- ‣ Left shift fills with 0 bits
- ‣ Right shift of unsigned variable fills with 0 bits
- ‣ Right shift of signed variable fills with sign bit (Actually implementation defined if negative!)

```
x << 2;  //  shifts bits of x two positions left
```
- ‣ Same as multiplying by 4

```
x >> 3;  //  shifts bits of x three positions right
```
- ‣ Same as dividing by 8 (if x is unsigned)

What does the following do?

```
x = ((x >> 2) << 2);
```

A. Changes x to be positive

B. Sets the least significant two bits to 0

C. Sets the most significant two bits to 0

D. Gives an integer overflow error

E. Implementation-defined behavior

Testing if a bit is set (i.e., is 1)

```c
#include <stdbool.h>

// Returns true if the nth bit of x is 1.
bool is_bit_set(unsigned int x, unsigned int n) {
  return x & (1u << n); // 1u is an unsigned int with value 1.
}
```

Testing if a bit is set (i.e., is 1)

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// Returns true if the nth bit of x is 1.
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}
```

`1u << n` gives an integer with only the nth bit set

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  return x & (1u << n); // 1u is an unsigned int with value 1.
}
```

`1u << n` gives an integer with only the nth bit set

If the *n*th bit is 1, then `x & (1u << n)` is `1u << n` which is nonzero.
If the *n*th bit is 0, then `x & (1u << n)` is 0

Testing if a bit is set (i.e., is 1)

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bool is_bit_set(unsigned int x, unsigned int n) {
  return x & (1u << n); // 1u is an unsigned int with value 1.
}
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If the *n*th bit is 1, then `x & (1u << n)` is `1u << n` which is nonzero.
If the *n*th bit is 0, then `x & (1u << n)` is 0

What happens if n is too large?

Testing if a bit is set (i.e., is 1)

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bool is_bit_set(unsigned int x, unsigned int n) {
  return x & (1u << n); // 1u is an unsigned int with value 1.
}
```

`1u << n` gives an integer with only the nth bit set

If the *n*th bit is 1, then `x & (1u << n)` is `1u << n` which is nonzero.
If the *n*th bit is 0, then `x & (1u << n)` is 0

What happens if n is too large?
‣ Undefined behavior!

UB

```c
#include <err.h>
#include <stdbool.h>
#include <stdio.h>
#include <stdlib.h>

// Returns true if the nth bit of x is 1.
bool is_bit_set(unsigned int x, unsigned int n) {
  return x & (1u << n);
}

int main(int argc, char **argv) {
  if (argc != 3)
    errx(1, "Usage: %s integer bit", argv[0]);
  unsigned int x = atoi(argv[1]);
  unsigned int n = atoi(argv[2]);
  if (is_bit_set(x, n))
    printf("Bit %u of %u is 1\n", n, x);
  else
    printf("Bit %u of %u is 0\n", n, x);
  return 0;
}
```

UB

`$ ./bad_shift 3 0`

```c
#include <err.h>
#include <stdbool.h>
#include <stdio.h>
#include <stdlib.h>

// Returns true if the nth bit of x is 1.
bool is_bit_set(unsigned int x, unsigned int n) {
  return x & (1u << n);
}

int main(int argc, char **argv) {
  if (argc != 3)
    errx(1, "Usage: %s integer bit", argv[0]);
  unsigned int x = atoi(argv[1]);
  unsigned int n = atoi(argv[2]);
  if (is_bit_set(x, n))
    printf("Bit %u of %u is 1\n", n, x);
  else
    printf("Bit %u of %u is 0\n", n, x);
  return 0;
}
```

UB

```
$ ./bad_shift 3 0
Bit 0 of 3 is 1
```

```c
#include <err.h>
#include <stdbool.h>
#include <stdio.h>
#include <stdlib.h>

// Returns true if the nth bit of x is 1.
bool is_bit_set(unsigned int x, unsigned int n) {
  return x & (1u << n);
}

int main(int argc, char **argv) {
  if (argc != 3)
    errx(1, "Usage: %s integer bit", argv[0]);
  unsigned int x = atoi(argv[1]);
  unsigned int n = atoi(argv[2]);
  if (is_bit_set(x, n))
    printf("Bit %u of %u is 1\n", n, x);
  else
    printf("Bit %u of %u is 0\n", n, x);
  return 0;
}
```

UB

```
$ ./bad_shift 3 0
Bit 0 of 3 is 1
$ ./bad_shift 3 1
```

```c
#include <err.h>
#include <stdbool.h>
#include <stdio.h>
#include <stdlib.h>

// Returns true if the nth bit of x is 1.
bool is_bit_set(unsigned int x, unsigned int n) {
  return x & (1u << n);
}

int main(int argc, char **argv) {
  if (argc != 3)
    errx(1, "Usage: %s integer bit", argv[0]);
  unsigned int x = atoi(argv[1]);
  unsigned int n = atoi(argv[2]);
  if (is_bit_set(x, n))
    printf("Bit %u of %u is 1\n", n, x);
  else
    printf("Bit %u of %u is 0\n", n, x);
  return 0;
}
```

UB

```
$ ./bad_shift 3 0
Bit 0 of 3 is 1
$ ./bad_shift 3 1
Bit 1 of 3 is 1
```

```c
#include <err.h>
#include <stdbool.h>
#include <stdio.h>
#include <stdlib.h>

// Returns true if the nth bit of x is 1.
bool is_bit_set(unsigned int x, unsigned int n) {
  return x & (1u << n);
}

int main(int argc, char **argv) {
  if (argc != 3)
    errx(1, "Usage: %s integer bit", argv[0]);
  unsigned int x = atoi(argv[1]);
  unsigned int n = atoi(argv[2]);
  if (is_bit_set(x, n))
    printf("Bit %u of %u is 1\n", n, x);
  else
    printf("Bit %u of %u is 0\n", n, x);
  return 0;
}
```

UB

```
$ ./bad_shift 3 0
Bit 0 of 3 is 1
$ ./bad_shift 3 1
Bit 1 of 3 is 1
$ ./bad_shift 3 2
```

```c
#include <err.h>
#include <stdbool.h>
#include <stdio.h>
#include <stdlib.h>

// Returns true if the nth bit of x is 1.
bool is_bit_set(unsigned int x, unsigned int n) {
  return x & (1u << n);
}

int main(int argc, char **argv) {
  if (argc != 3)
    errx(1, "Usage: %s integer bit", argv[0]);
  unsigned int x = atoi(argv[1]);
  unsigned int n = atoi(argv[2]);
  if (is_bit_set(x, n))
    printf("Bit %u of %u is 1\n", n, x);
  else
    printf("Bit %u of %u is 0\n", n, x);
  return 0;
}
```

UB

```
$ ./bad_shift 3 0
Bit 0 of 3 is 1
$ ./bad_shift 3 1
Bit 1 of 3 is 1
$ ./bad_shift 3 2
Bit 2 of 3 is 0
```

```c
#include <err.h>
#include <stdbool.h>
#include <stdio.h>
#include <stdlib.h>

// Returns true if the nth bit of x is 1.
bool is_bit_set(unsigned int x, unsigned int n) {
  return x & (1u << n);
}

int main(int argc, char **argv) {
  if (argc != 3)
    errx(1, "Usage: %s integer bit", argv[0]);
  unsigned int x = atoi(argv[1]);
  unsigned int n = atoi(argv[2]);
  if (is_bit_set(x, n))
    printf("Bit %u of %u is 1\n", n, x);
  else
    printf("Bit %u of %u is 0\n", n, x);
  return 0;
}
```

UB

```
$ ./bad_shift 3 0
Bit 0 of 3 is 1
$ ./bad_shift 3 1
Bit 1 of 3 is 1
$ ./bad_shift 3 2
Bit 2 of 3 is 0
$ ./bad_shift 3 32
```

```c
#include <err.h>
#include <stdbool.h>
#include <stdio.h>
#include <stdlib.h>

// Returns true if the nth bit of x is 1.
bool is_bit_set(unsigned int x, unsigned int n) {
  return x & (1u << n);
}


int main(int argc, char **argv) {
  if (argc != 3)
    errx(1, "Usage: %s integer bit", argv[0]);
  unsigned int x = atoi(argv[1]);
  unsigned int n = atoi(argv[2]);
  if (is_bit_set(x, n))
    printf("Bit %u of %u is 1\n", n, x);
  else
    printf("Bit %u of %u is 0\n", n, x);
  return 0;
}
```

UB

```
$ ./bad_shift 3 0
Bit 0 of 3 is 1
$ ./bad_shift 3 1
Bit 1 of 3 is 1
$ ./bad_shift 3 2
Bit 2 of 3 is 0
$ ./bad_shift 3 32
Bit 32 of 3 is 1
```

```c
#include <err.h>
#include <stdbool.h>
#include <stdio.h>
#include <stdlib.h>

// Returns true if the nth bit of x is 1.
bool is_bit_set(unsigned int x, unsigned int n) {
    return x & (1u << n);
}

int main(int argc, char **argv) {
    if (argc != 3)
        errx(1, "Usage: %s integer bit", argv[0]);
    unsigned int x = atoi(argv[1]);
    unsigned int n = atoi(argv[2]);
    if (is_bit_set(x, n))
        printf("Bit %u of %u is 1\n", n, x);
    else
        printf("Bit %u of %u is 0\n", n, x);
    return 0;
}
```

UB

```
$ ./bad_shift 3 0
Bit 0 of 3 is 1
$ ./bad_shift 3 1
Bit 1 of 3 is 1
$ ./bad_shift 3 2
Bit 2 of 3 is 0
$ ./bad_shift 3 32
Bit 32 of 3 is 1
$ ./bad_shift 3 33
```

```c
#include <err.h>
#include <stdbool.h>
#include <stdio.h>
#include <stdlib.h>

// Returns true if the nth bit of x is 1.
bool is_bit_set(unsigned int x, unsigned int n) {
    return x & (1u << n);
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int main(int argc, char **argv) {
    if (argc != 3)
        errx(1, "Usage: %s integer bit", argv[0]);
    unsigned int x = atoi(argv[1]);
    unsigned int n = atoi(argv[2]);
    if (is_bit_set(x, n))
        printf("Bit %u of %u is 1\n", n, x);
    else
        printf("Bit %u of %u is 0\n", n, x);
    return 0;
}
```

UB

```
$ ./bad_shift 3 0
Bit 0 of 3 is 1
$ ./bad_shift 3 1
Bit 1 of 3 is 1
$ ./bad_shift 3 2
Bit 2 of 3 is 0
$ ./bad_shift 3 32
Bit 32 of 3 is 1
$ ./bad_shift 3 33
Bit 33 of 3 is 1
```

```c
#include <err.h>
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// Returns true if the nth bit of x is 1.
bool is_bit_set(unsigned int x, unsigned int n) {
  return x & (1u << n);
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int main(int argc, char **argv) {
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  else
    printf("Bit %u of %u is 0\n", n, x);
  return 0;
}
```

UB

```
$ ./bad_shift 3 0
Bit 0 of 3 is 1
$ ./bad_shift 3 1
Bit 1 of 3 is 1
$ ./bad_shift 3 2
Bit 2 of 3 is 0
$ ./bad_shift 3 32
Bit 32 of 3 is 1
$ ./bad_shift 3 33
Bit 33 of 3 is 1
$ ./bad_shift 3 34
```

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#include <err.h>
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  if (is_bit_set(x, n))
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  else
    printf("Bit %u of %u is 0\n", n, x);
  return 0;
}
```

UB

```
$ ./bad_shift 3 0
Bit 0 of 3 is 1
$ ./bad_shift 3 1
Bit 1 of 3 is 1
$ ./bad_shift 3 2
Bit 2 of 3 is 0
$ ./bad_shift 3 32
Bit 32 of 3 is 1
$ ./bad_shift 3 33
Bit 33 of 3 is 1
$ ./bad_shift 3 34
Bit 34 of 3 is 0
```

```c
#include <err.h>
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// Returns true if the nth bit of x is 1.
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int main(int argc, char **argv) {
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    if (is_bit_set(x, n))
        printf("Bit %u of %u is 1\n", n, x);
    else
        printf("Bit %u of %u is 0\n", n, x);
    return 0;
}
```

Testing if a bit is set (i.e., is 1)

```c
#include <assert.h>
#include <limits.h>
#include <stdbool.h>

// Returns true if the nth bit of x is 1.
bool is_bit_set(unsigned int x, unsigned int n) {
  // assert(cond) will abort at runtime if cond is false.
  assert(n < CHAR_BIT * sizeof x);
  return x & (1u << n); // 1u is an unsigned int with value 1.
}
```

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  assert(n < CHAR_BIT * sizeof x);
  return x & (1u << n); // 1u is an unsigned int with value 1.
}
```

E.g., if **CHAR_BIT** is 8 and **sizeof** x is 4, then `n` must be less than 32 or the program aborts

Setting a bit (to 1)

```c
// Returns the value of x with the nth bit set to 1.
unsigned int set_bit(unsigned int x, unsigned int n) {
    assert(n < CHAR_BIT * sizeof x);
    return x | (1u << n);
}
```

Clearing a bit (setting it to 0)

```c
// Returns the value of x with the nth bit set to 0.
unsigned int set_bit(unsigned int x, unsigned int n) {
  assert(n < CHAR_BIT * sizeof x);
  return x & ~(1u << n);
}
```

Clearing a bit (setting it to 0)

```c
// Returns the value of x with the nth bit set to 0.
unsigned int set_bit(unsigned int x, unsigned int n) {
  assert(n < CHAR_BIT * sizeof x);
  return x & ~(1u << n);
}
```

`1u << n` gives an integer with just the nth bit set

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// Returns the value of x with the nth bit set to 0.
unsigned int set_bit(unsigned int x, unsigned int n) {
    assert(n < CHAR_BIT * sizeof x);
    return x & ~(1u << n);
}
```

`1u << n` gives an integer with just the nth bit set

`~(1u << n)` gives an integer with all bits set *except* the nth bit

Given an unsigned integer `x` with some value, what value should we use for `mask` to clear all of the bits of `x` except for the least significant 5 bits?

```cpp
unsigned int x = /* … */;      // Given some value here,
unsigned int mask = /* … */;   // what value goes here
x = x & mask;                  // to clear the required bits?
```

A. `0x5u`

B. `~0x5u`

C. `0x1Fu`

D. `~0x1Fu`

E. `sizeof x – 5`

Given an unsigned integer `x` with some value, what value should we use for `mask` to clear the 5 least significant bits of `x`?

```
unsigned int x = /* … */;      // Given some value here,
unsigned int mask = /* … */;   // what value goes here
x = x & mask;                  // to clear the required bits?
```

A. 0x5u

B. ~0x5u

C. 0x1Fu

D. ~0x1Fu

E. sizeof x – 5

Combining flags via |

Specify flags via individual bits

Combine flags with |

E.g., set file system
permissions via the flags
`S_I{R,W,X}{USR,GRP,OTH}`

```c
#define S_IRWXU 0000700      /* RWX mask for owner */
#define S_IRUSR 0000400      /* R for owner */
#define S_IWUSR 0000200      /* W for owner */
#define S_IXUSR 0000100      /* X for owner */

#define S_IRWXG 0000070      /* RWX mask for group */
#define S_IRGRP 0000040      /* R for group */
#define S_IWGRP 0000020      /* W for group */
#define S_IXGRP 0000010      /* X for group */

#define S_IRWXO 0000007      /* RWX mask for other */
#define S_IROTH 0000004      /* R for other */
#define S_IWOTH 0000002      /* W for other */
#define S_IXOTH 0000001      /* X for other */

int chmod(char const *path, mode_t mode);
```

Negative numbers

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Usually stored using two's complement
- ‣ Take the magnitude of the number
- ‣ Invert all of the bits
- ‣ Add 1

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representation of -5 in 8 bits

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representation of -5 in 8 bits
```
magnitude:    0000_0101
```

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representation of -5 in 8 bits
```
magnitude:    0000_0101
invert bits: 1111_1010
```

Negative numbers

Usually stored using two's complement
- ‣ Take the magnitude of the number
- ‣ Invert all of the bits
- ‣ Add 1

representation of -5 in 8 bits

```
magnitude:    0000_0101
invert bits: 1111_1010
Add 1:       1111_1011
```

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Usually stored using two's complement
- ‣ Take the magnitude of the number
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Computing -x from x (regardless of sign)
- ‣ Invert all of the bits
- ‣ Add 1

representation of -5 in 8 bits
```
magnitude:    0000_0101
invert bits: 1111_1010
Add 1:       1111_1011
```

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representation of -5 in 8 bits
```
magnitude:    0000_0101
invert bits: 1111_1010
Add 1:        1111_1011
```

-(-5) in 8 bits

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representation of -5 in 8 bits
```
magnitude:    0000_0101
invert bits: 1111_1010
Add 1:       1111_1011
```

-(-5) in 8 bits
```
-5:          1111_1011
```

Negative numbers

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- ‣ Take the magnitude of the number
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representation of -5 in 8 bits
```
magnitude:    0000_0101
invert bits: 1111_1010
Add 1:        1111_1011
```

-(-5) in 8 bits
```
-5:           1111_1011
invert bits: 0000_0100
```

Negative numbers

Usually stored using two's complement
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```
representation of -5 in 8 bits
magnitude:    0000_0101
invert bits: 1111_1010
Add 1:       1111_1011

-(-5) in 8 bits
-5:          1111_1011
invert bits: 0000_0100
Add 1:       0000_0101
```

Negative numbers

Usually stored using two's complement
‣ Take the magnitude of the number
‣ Invert all of the bits
‣ Add 1

Computing -x from x (regardless of sign)
‣ Invert all of the bits
‣ Add 1

representation of -5 in 8 bits
```
magnitude:    0000_0101
invert bits: 1111_1010
Add 1:        1111_1011
```

-(-5) in 8 bits
```
-5:           1111_1011
invert bits: 0000_0100
Add 1:        0000_0101
```
```
0:            0000_0000
```

Negative numbers

Usually stored using two's complement
- ‣ Take the magnitude of the number
- ‣ Invert all of the bits
- ‣ Add 1

Computing -x from x (regardless of sign)
- ‣ Invert all of the bits
- ‣ Add 1

```
representation of -5 in 8 bits
magnitude:    0000_0101
invert bits:  1111_1010
Add 1:        1111_1011

-(-5) in 8 bits
-5:           1111_1011
invert bits:  0000_0100
Add 1:        0000_0101

0:            0000_0000
invert bits:  1111_1111
```

Negative numbers

Usually stored using two's complement
- ‣ Take the magnitude of the number
- ‣ Invert all of the bits
- ‣ Add 1

Computing -x from x (regardless of sign)
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-(-5) in 8 bits
-5:            1111_1011
invert bits:   0000_0100
Add 1:         0000_0101


0:             0000_0000
invert bits:   1111_1111
Add 1:        1_0000_0000
```

Negative numbers

Usually stored using two's complement
- ‣ Take the magnitude of the number
- ‣ Invert all of the bits
- ‣ Add 1

Computing -x from x (regardless of sign)
- ‣ Invert all of the bits
- ‣ Add 1

Most significant bit indicates the sign
- ‣ 1 indicates a negative number

```
representation of -5 in 8 bits
magnitude:     0000_0101
invert bits:   1111_1010
Add 1:         1111_1011

-(-5) in 8 bits
-5:            1111_1011
invert bits:   0000_0100
Add 1:         0000_0101

0:             0000_0000
invert bits:   1111_1111
Add 1:         1_0000_0000
```

Signed numbers in two's complement

```
10000000 00000000 00000000 00000000 = –2³¹
10000000 00000000 00000000 00000001 = –2³¹+1

…
11111111 11111111 11111111 11111110 = –2
11111111 11111111 11111111 11111111 = –1
00000000 00000000 00000000 00000000 = 0
00000000 00000000 00000000 00000001 = 1
00000000 00000000 00000000 00000010 = 2
00000000 00000000 00000000 00000011 = 3

…
01111111 11111111 11111111 11111110 = 2³¹–2
01111111 11111111 11111111 11111111 = 2³¹–1
```

Not the only choice

Not the only choice

Sign and magnitude
- ‣ Most significant bit represents the sign, remaining bits are the magnitude
- ‣ Range $-(2^{n-1} - 1)$ to $2^{n-1} - 1$
- ‣ Two different bit patterns for zero: 0 and 0x80000000 (assuming 32-bits)

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- ‣ Most significant bit represents the sign, remaining bits are the magnitude
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- ‣ Two different bit patterns for zero: 0 and 0x80000000 (assuming 32-bits)

Ones' complement
- ‣ Negative numbers are the bitwise inverse of positive numbers (-x = ~x)
- ‣ Range $-(2^{n-1} - 1)$ to $2^{n-1} - 1$
- ‣ Two different bit patterns for zero: 0 and 0xFFFFFFFF (assuming 32-bits)

Not the only choice

Sign and magnitude
- ‣ Most significant bit represents the sign, remaining bits are the magnitude
- ‣ Range $-(2^{n-1} - 1)$ to $2^{n-1} - 1$
- ‣ Two different bit patterns for zero: 0 and 0x80000000 (assuming 32-bits)

Ones' complement
- ‣ Negative numbers are the bitwise inverse of positive numbers ($-x = \sim x$)
- ‣ Range $-(2^{n-1} - 1)$ to $2^{n-1} - 1$
- ‣ Two different bit patterns for zero: 0 and 0xFFFFFFFF (assuming 32-bits)

Two's complement
- ‣ Negative numbers are ones' complement plus one ($-x = \sim x + 1$)
- ‣ Range -2^{n-1} to $2^{n-1} - 1$
- ‣ Only one zero

In-class exercise

https://checkoway.net/teaching/cs241/2020-spring/exercises/Lecture-13.html

Grab a laptop and a partner and try to get as much of that done as you can!