

CS 241: Systems Programming

Lecture 12. Bits and Bytes 1

Fall 2019

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Computers use binary

Everything in a computer is stored and manipulated as a collection of bits

- ▶ The bits mean something only in how they are used, **not** what they are

Example with 32-bits: 01000011010100110100001101001001

- ▶ As a integer: 1129530185
- ▶ As a (single-precision) floating point number: 211.262833
- ▶ As a sequence of four ASCII characters: CSCI
- ▶ As 32-bit x86 instructions:

```
inc    ebx
```

```
push  ebx
```

```
inc    ebx
```

```
dec    ecx
```

Base 10 review

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Given a decimal (base 10) number 1253_{10}

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- ▶ 3 ones (10^0)
- ▶ 5 tens (10^1)
- ▶ 2 hundreds (10^2)
- ▶ 1 thousand (10^3)

10^3	10^2	10^1	10^0
1	2	5	3

Base 8 (Octal)

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Only uses the digits 0-7

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Given a octal (base 8) number 02345

- ▶ 5 ones (8^0)
- ▶ 4 eights (8^1)
- ▶ 3 sixty-fours (8^2)
- ▶ 2 five hundred twelves (8^3)

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2	3	4	5

Base 16 (Hexadecimal)

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Single place has values of 0-15

- ▶ Need digits larger than 9. Use A=10, B=11, ..., F=15
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Given a hexadecimal (base 16) number `0x04E5`

- ▶ 5 ones (16^0)
- ▶ 14 sixteens (16^1)
- ▶ 4 two hundred fifty-sixes (16^2)
- ▶ 0 four thousand ninety-sixes (16^3)

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Base 2 (Binary)

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Given a binary number 0b0000010011100101

- ▶ $1 \cdot 2^0$, $0 \cdot 2^1$, $1 \cdot 2^2$, $0 \cdot 2^3$
- ▶ $0 \cdot 2^4$, $1 \cdot 2^5$, $1 \cdot 2^6$, $1 \cdot 2^7$
- ▶ $0 \cdot 2^8$, $0 \cdot 2^9$, $1 \cdot 2^{10}$, $0 \cdot 2^{11}$
- ▶ $0 \cdot 2^{12}$, $0 \cdot 2^{13}$, $0 \cdot 2^{14}$, $0 \cdot 2^{15}$

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- ▶ $0 \cdot 2^8, 0 \cdot 2^9, 1 \cdot 2^{10}, 0 \cdot 2^{11}$
- ▶ $0 \cdot 2^{12}, 0 \cdot 2^{13}, 0 \cdot 2^{14}, 0 \cdot 2^{15}$

$2^{15..12}$ $2^{11..8}$ $2^{7..4}$ $2^{3..0}$

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- ▶ $0 \cdot 2^4$, $1 \cdot 2^5$, $1 \cdot 2^6$, $1 \cdot 2^7$
- ▶ $0 \cdot 2^8$, $0 \cdot 2^9$, $1 \cdot 2^{10}$, $0 \cdot 2^{11}$
- ▶ $0 \cdot 2^{12}$, $0 \cdot 2^{13}$, $0 \cdot 2^{14}$, $0 \cdot 2^{15}$

$2^{15..12}$	$2^{11..8}$	$2^{7..4}$	$2^{3..0}$
0000	0100	1110	0101

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▶ $1253 = 1*10^3 + 2*10^2 + 5*10^1 + 3*10^0 = 1253$

▶ $02345 = 2*8^3 + 3*8^2 + 4*8^1 + 5*8^0 = 1253$

▶ $0x04E5 = 0*16^3 + 4*16^2 + 14*16^1 + 5*16^0 = 1253$

▶ $0b0000010011100101 = 1253$

Convert the octal value 031 to decimal

A. 7

B. 25

C. 31

D. 49

E. 248

Converting binary to hex

Hex	Binary	Hex	Binary
0	0000	8	1000
1	0001	9	1001
2	0010	A	1010
3	0011	B	1011
4	0100	C	1100
5	0101	D	1101
6	0110	E	1110
7	0111	F	1111

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Just group digits by 4s starting with LSB

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Converting binary to octal

Octal	Binary
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

Converting binary to octal

Just group digits by 3s starting with LSB

Octal	Binary
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

Converting binary to octal

Just group digits by 3s starting with LSB

▶ `0b0000010011100101`

Octal	Binary
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

Converting binary to octal

Just group digits by 3s starting with LSB

- ▶ 0b0000010011100101
- ▶ 0b 000 000 010 011 100 101

Octal	Binary
0	000
1	001
2	010
3	011
4	100
5	101
6	110
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Converting binary to octal

Just group digits by 3s starting with LSB

- ▶ 0b0000010011100101
- ▶ 0b 000 000 010 011 100 101

Each block of 3 bits is 0–7

Octal	Binary
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

Converting binary to octal

Just group digits by 3s starting with LSB

- ▶ 0b0000010011100101
- ▶ 0b 000 000 010 011 100 101

Each block of 3 bits is 0–7

- ▶ Replace each with a octal digit

Octal	Binary
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

Converting binary to octal

Just group digits by 3s starting with LSB

- ▶ 0b0000010011100101
- ▶ 0b 000 000 010 011 100 101

Each block of 3 bits is 0–7

- ▶ Replace each with a octal digit
- ▶ 0 0 2 3 4 5

Octal	Binary
0	000
1	001
2	010
3	011
4	100
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6	110
7	111

Converting binary to octal

Just group digits by 3s starting with LSB

- ▶ 0b0000010011100101
- ▶ 0b 000 000 010 011 100 101

Each block of 3 bits is 0–7

- ▶ Replace each with a octal digit
- ▶ 0 0 2 3 4 5
- ▶ 0002345 (We prepended a 0 to denote octal)

Octal	Binary
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

Convert the 16-bit binary number 0b11001010_11111110 to hex.

(I added an underscore to separate the two groups of 8 bits to improve readability.)

A. 0xBEEF

B. 0xCAFE

C. 0xDEAD

D. 0xFACE

E. 0xFEEF

Hex	Binary	Hex	Binary
-----	--------	-----	--------

0	0000	8	1000
---	------	---	------

1	0001	9	1001
---	------	---	------

2	0010	A	1010
---	------	---	------

3	0011	B	1011
---	------	---	------

4	0100	C	1100
---	------	---	------

5	0101	D	1101
---	------	---	------

6	0110	E	1110
---	------	---	------

7	0111	F	1111
---	------	---	------

Alternative view of hex/octal

Binary is a pain to read/work with

- ▶ Consider a 64-bit number

```
0b010011000100000011010110101100000011011011000101010  
0011110110000
```

so long it doesn't fit on one line!

Hex (and much less commonly octal) can be viewed as a much more compact way to represent binary numbers

- ▶ **0x4c40d6b036c547b0**

Converting hex/octal to binary

1. Take each hexadecimal (or octal) digit
2. Convert it into binary
 - ▶ 4 places hex (e.g., A becomes 1010)
 - ▶ 3 places octal (e.g., 6 becomes 110)
3. Group them together from LSB to MSB

Converting between Hex & Octal

1. Take hexadecimal number
2. Convert to binary
3. Regroup in clusters of 3 from LSB
4. Generate Octal digits
5. Use reverse process for Octal to Hex

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Repeatedly divide by 2, recording remainders

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Example: 39

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$$\blacktriangleright 39 / 2 = 19 \text{ r } 1$$

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Example: 39

$$\blacktriangleright 39 / 2 = 19 \text{ r } 1$$

$$\blacktriangleright 19 / 2 = 9 \text{ r } 1$$

Converting decimal to binary

Repeatedly divide by 2, recording remainders

Remainders form the binary number from least to most significant

Example: 39

$$\blacktriangleright 39 / 2 = 19 \text{ r } 1$$

$$\blacktriangleright 19 / 2 = 9 \text{ r } 1$$

$$\blacktriangleright 9 / 2 = 4 \text{ r } 1$$

Converting decimal to binary

Repeatedly divide by 2, recording remainders

Remainders form the binary number from least to most significant

Example: 39

- ▶ $39 / 2 = 19 \text{ r } 1$
- ▶ $19 / 2 = 9 \text{ r } 1$
- ▶ $9 / 2 = 4 \text{ r } 1$
- ▶ $4 / 2 = 2 \text{ r } 0$

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Example: 39

- ▶ $39 / 2 = 19 \text{ r } 1$
- ▶ $19 / 2 = 9 \text{ r } 1$
- ▶ $9 / 2 = 4 \text{ r } 1$
- ▶ $4 / 2 = 2 \text{ r } 0$
- ▶ $2 / 2 = 1 \text{ r } 0$

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- ▶ $19 / 2 = 9 \text{ r } 1$
- ▶ $9 / 2 = 4 \text{ r } 1$
- ▶ $4 / 2 = 2 \text{ r } 0$
- ▶ $2 / 2 = 1 \text{ r } 0$
- ▶ $1 / 2 = 0 \text{ r } 1$

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Example: 39

- ▶ $39 / 2 = 19 \text{ r } 1$
- ▶ $19 / 2 = 9 \text{ r } 1$
- ▶ $9 / 2 = 4 \text{ r } 1$
- ▶ $4 / 2 = 2 \text{ r } 0$
- ▶ $2 / 2 = 1 \text{ r } 0$
- ▶ $1 / 2 = 0 \text{ r } 1$
- ▶ $39 = 0b100111$

In-class exercise

<https://checkoway.net/teaching/cs241/2019-fall/exercises/Lecture-12.html>

Grab a laptop and a partner and try to get as much of that done as you can!