CS 383: Theory of Computation **Spring 2024**

Homework 2

Due: Friday, February 23, 2024

Instructions

For each of the following problems, construct a DFA **in JFLAP** that recognizes the given language. Each DFA should be in its own file. **Each DFA must be a valid DFA! This means they must have one initial state, every state must have a transition for each symbol in the alphabet, and no states may have multiple transitions for the same alphabet symbol. JFLAP does not enforce this.** The name of each file should be the problem number with the `.jff` extension: `1.jff`, `2.jff`, and so on.

Upload your solutions to GradeScope by dragging all of the `.jff` files onto the website. **IMPORTANT: If you do not have the files named exactly as specified, the autograded portion will not work. If you only upload a single file, the autograder will not work.** (If you wish to upload just a single file, for example, `1.jff`, you'll also need to upload some other file such as a blank text file. This is a limitation of GradeScope.) Not all autograded tests will be visible prior to the deadline.

Problem 1 Construct a DFA that recognizes the language $A = \{w \mid w$ starts with `a` and has odd length or starts with `b` and has even length$\}$. The alphabet is $\Sigma = \{\texttt{a}, \texttt{b}\}$.

Problem 2 Construct a DFA that recognizes the language $B = \{w \mid w$ does not contain the substring `ab` and does not contain the substring `ba`$\}$. The alphabet is $\Sigma = \{\texttt{a}, \texttt{b}\}$.

Problem 3 Construct a DFA that recognizes the language $C = \{w|\ w$ is a binary number divisible by 5$\}$. For example, `1001` (9 in base 10) is not divisible by 5 so it would be rejected, whereas `1010` (10 in base 10) *is* divisible by 5 so it would be accepted. Leading `0`'s (e.g., `001010`) are permitted and the empty string is taken as a representation for `0`. The alphabet is $\Sigma = \{\texttt{0}, \texttt{1}\}$.

[*Hint: Let the states represent the number read so far, modulo 5. Each new input symbol that's read causes the existing number to be multiplied by 2 and then the input symbol is added. For example, if the DFA has read the string* `110` *(decimal value 6), and the next input symbol is a* `1`*, then the the string read so far at this point,* `1101` *has decimal value* $6 \cdot 2 + 1 = 13$*. Furthermore, note that* $6 \bmod 5 = 1$*,* $13 \bmod 5 = 3$*, and* $((6 \bmod 5) \cdot 2 + 1) \bmod 5 = 3$*. Use this observation to construct the transitions.*]

Problem 4 Construct a DFA that recognizes the language $D = \{w \mid w$ contains exactly three `a` symbols and at least two `b` symbols$\}$. The alphabet is $\Sigma = \{\texttt{a}, \texttt{b}\}$.

[*Hint: Construct a DFA for the language* $D_1 = \{w \mid w$ *contains exactly three* `a` *symbols*$\}$ *and construct a DFA for the language* $D_2 = \{w \mid w$ *contains at least two* `b` *symbols*$\}$. *Use the product construction described in the book to build a DFA that recognizes* $D = D_1 \cap D_2$.]